Joseph Pedlosky

# Geophysical Fluid Dynamics



Springer-Verlag
New York Heidelberg Berlin

## Joseph Pedlosky

## Geophysical Fluid Dynamics



### **Contents**

1.1	geophysical Fluid Dynamics 1		
1.2	The Rossby Number 2		
1.3 I	Density Stratification 8		
1.4	The Equations of Motion in a Nonrotating Coordinate Frame	10	
1.5 I	Rotating Coordinate Frames 14		
1.6 E	Equations of Motion in a Rotating Coordinate Frame 17		
1.7	Coriolis Acceleration and the Rossby Number 20		
Fund	damentals		22
2.1	Vorticity 22		
2.2	The Circulation 28		
2.3	Kelvin's Theorem 33		
2.4	The Vorticity Equation 34		
2.5	Potential Vorticity 38		
2.6	The Thermal Wind 42		
2.7	The Taylor-Proudman Theorem 43		
2.8	Geostrophic Motion 45		
2.9	Consequences of the Geostrophic and Hydrostatic		
	Approximations 51		
2.10	Geostrophic Degeneracy 55		

3	Inviscid Shallow-Water Theory	57
	3.1 Introduction 57	
	3.2 The Shallow-Water Model 58	
	3.3 The Shallow-Water Equations 59	
	3.4 Potential-Vorticity Conservation: Shallow-Water Theory 63	
	3.5 Integral Constraints 65	
	3.6 Small-Amplitude Motions 67	
	3.7 Linearized Geostrophic Motion 69	
	3.8 Plane Waves in a Layer of Constant Depth 71	
	3.9 Poincaré and Kelvin Waves 75	
	3.10 The Rossby Wave 81	
	3.11 Dynamic Diagnosis of the Rossby Wave 84	
	3.12 Quasigeostrophic Scaling in Shallow-Water Theory 86	
	3.13 Steady Quasigeostrophic Motion 93	
	3.14 Inertial Boundary Currents 94	
	3.15 Quasigeostrophic Rossby Waves 99	
	3.16 The Mechanism for the Rossby Wave 102	
	3.17 The Beta-Plane 105	
	3.18 Rossby Waves in a Zonal Current 108	
	3.19 Group Velocity 111	
	3.20 The Method of Multiple Time Scales 118	
	3.21 Energy and Energy Flux in Rossby Waves 121	
	3.22 The Energy Propagation Diagram 123	
	3.23 Reflection and the Radiation Condition 124	
	3.24 Rossby Waves Produced by an Initial Disturbance 130	
	3.25 Quasigeostrophic Normal Modes in Closed Basins 144	
	3.26 Resonant Interactions 153	
	3.27 Energy and Enstrophy 164	
	Appendix to Chapter 3 166	
	,	
4	Friction and Viscous Flow	168
	4.1 Introduction 168	
	4.2 Turbulent Reynolds Stresses 170	
	4.3 The Ekman Layer 174	

#### 4.4 The Nature of Nearly Frictionless Flow 183 4.5 Boundary-Layer Theory 189 4.6 Quasigeostrophic Dynamics in the Presence of Friction 201 4.7 Spin-Down 205 4.8 Steady Motion 206 4.9 Ekman Layer on a Sloping Surface 208 4.10 Ekman Layer on a Free Surface 215 4.11 Quasigeostrophic Potential Vorticity Equation with Friction and Topography 222

4.12 The Decay of a Rossby Wave 225 4.13 Side-Wall Friction Layers 227

5	Hon	nogeneous Models of the Wind-Driven Oceanic Circulation	236
	5.1	Introduction 236	
	5.2		
	5.3		
	5.4	The control of the co	
	5.5	Stommel's Model: Bottom Friction Layer 264	
	5.6	Inertial Boundary-Layer Theory 270	
	5.7		
	5.8		anic
	5.9	Dissipation Integrals for Steady Circulations 281	
	5.10	Free Inertial Modes 287	
	5.11	Numerical Experiments 290	
		Ekman Upwelling Circulations 297	
		The Effect of Bottom Topography 308	
		Concluding Remarks on the Homogeneous Model 313	
6	Qua	sigeostrophic Motion of a Stratified Fluid on a Sphere	314
	6.1	Introduction 314	
	6.2		315
	6.3	Geostrophic Approximation: $\varepsilon = O(L/r_0) \ll 1$ 323	
	6.4	The Concept of Static Stability 329	
	6.5	Quasigeostrophic Potenial-Vorticity Equation for Atmospheric	
	0.5	Synoptic Scales 333	
	6.6	The Ekman Layer in a Stratified Fluid 338	
	6.7	Boundary Conditions for the Potential Vorticity Equation:	The
	01,	Atmosphere 340	THO
	6.8	Quasigeostrophic Potential-Vorticity Equation for Oceanic Synoptic Scales 340	
	6.9	Boundary Conditions for the Potential-Vorticity Equation: Oceans 343	the
	6.10	Geostrophic Energy Equation and Available Potential Energy	346
		Rossby Waves in a Stratified Fluid 352	
	6.12	Rossby-Wave Normal Modes: the Vertical Structure Equation	356
	6.13	Forced Stationary Waves in the Atmosphere 363	
	6.14	Wave-Zonal-Flow Interaction Theorems 371	
	6.15	Topographic Waves in a Stratified Ocean 378	
	6.16	Layer Models 386	
	6.17	Rossby Waves in the Two-Layer Model 394	
		reaction and the same to the control of the same of the control of the same of the control of th	396
		Geostrophic Approximation $\varepsilon \ll L/r_0 < 1$ ; the Sverdrup	
		Relation 400	
	6.20	Geostrophic Approximation $\varepsilon \ll 1$ , $L/r_0 = O(1)$ 404	
		The Thermocline Problem 409	

7

**Instability Theory** 

7.1 Introduction 423

Stratified Model 426

	7.4 Normal Modes 441	
	7.5 Bounds on the Phase Speed and Growth Rate 447	
	7.6 Baroclinic Instability: the Basic Mechanism 451	
	7.7 Eady's Model 456	
	7.8 Charney's Model and Critical Layers 465	
	7.9 Instability in the Two-Layer Model: Formulation 477	
	7.10 Normal Modes in the Two-Layer Model: Necessary Condition for Instability 481	ons
	7.11 Baroclinic Instability in the Two-Layer Model: Phillips' Model	485
	7.12 Effects of Friction 492	
	7.13 Baroclinic Instability of Nonzonal Flows 497	
	7.14 Barotropic Instability 504	
	7.15 Instability of Currents with Horizontal and Vertical Shear 512	
	7.16 Nonlinear Theory of Baroclinic Instability 519	
8	Ageostrophic Motion	540
	8.1 Anisotropic Scales 540	
	8.2 Continental-Shelf Waves 544	
	<ul><li>8.3 Slow Circulation of a Stratified, Dissipative Fluid 553</li><li>8.4 The Theory of Frontogenesis 569</li><li>8.5 Equatorial Waves 586</li></ul>	
	Selected Bibliography	605
	Index	619

7.2 Formulation of the Instability Problem: The Continuously

7.3 The Linear Stability Problem: Conditions for Instability 432

423